Time Dispersion Properties of Cascaded Multimode Fiber Links

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Abstract—A general model for studying the time dispersion properties of cascaded multimode fiber links is presented. Fibers having different index profiles and misaligned joints are taken into account. It is shown that the transit time of a ray in the first fiber is conserved and transferred to another ray in the second fiber, and so forth. Simple compensation formulas are derived for α-profile fibers. The lower limit to the total time dispersion, which is imposed by material dispersion, is also investigated.

I. Introduction

It is well known [1]–[5] that cascaded multimode fibers sometimes exhibit a compensation of their time dispersion properties. This is due to their index profiles, which may be either undercompensated or overcompensated. Nevertheless, this compensation should be considered with some cares, in fact it affects only intermodal time dispersion (intramodal time dispersion remains unchanged). Moreover, distributed mode coupling tends to reduce any compensation effect, although it may be beneficial for the overall fiber bandwidth [6], [7]. Finally, considering the very critical role played by joint misalignment on the time dispersion properties of a pair of equal near-parabolic fibers recently shown in [8], we can expect that joint misalignment affects also the compensation behavior of two or more cascaded fibers.

In this paper we study the time dispersion properties of an optical link, made of different cascaded multimode fibers, in the presence of material dispersion and joint misalignments. Distributed mode coupling is ignored here, for the sake of simplicity, and will be considered in a subsequent paper. However, the present approach can be considered valid for practical fibers, provided that they are loosely jacketed. The theoretical model is based on ray optics, which allow a simple characterization of the joint. Ray optics give acceptable results only when several hundred modes are propagating. Such a condition is, however, well satisfied in usual multimode fibers.

In order to obtain general results, we do not consider a specific dopant for the profile fabrication (e.g., Ge, P, $\cdots$), which would be characterized by a particular material dispersion curve. Nevertheless, assuming an optical source whose power spectrum $p(\lambda)$ is centered at the wavelength λ_o and has a linewidth $\Delta\lambda$, we will consider some behavior limits on the baseband response due to intramodal time dispersion, which depend on $\Delta\lambda$ and on the parameter $d\tau/d\lambda|_{\lambda=\lambda_o}$, where τ is the time delay per unit distance of the chromatic component characterized by the wavelength λ. In this way, we can predict when intramodal time dispersion becomes predominant over intermodal time dispersion, making profile compensation useless.

There are different kinds of joint misalignments [9], but probably the most frequent and critical, for both attenuation and time dispersion properties, is a lateral displacement, sometimes called offset. In [10], it has been shown that also an angular misalignment (tilt) can be regarded as an equivalent offset. Letting d be the lateral displacement between the two fiber axes, and a their core radii, the offset can be measured by the normalized quantity d/a. Only very small values of d/a will be considered in the following, since it is typically some parts per thousand in fusion splices, whereas it may reach few parts per hundred in mechanical splices or in demountable connectors.

A very wide class of monotonic graded-index profile distributions is well approximated by a so-called α-profile distribution [11]. This is characterized by the property that the group delay per unit distance of any ray, and even the optical power carried by that ray under a uniform excitation, depend only on its propagation constant β. This feature much simplifies the theoretical treatment of the problem. Nevertheless, recently [5], some limits of the α-profile to reproduce actual fiber index profiles have been stressed. So, we proceed as follows: firstly we consider α-profile fibers, which have the advantage of a simpler formalism, and then we take into account the case of a general profile, giving some formal relations.

II. General Principles of the Theoretical Model

For α-profile fibers, letting x be a suitably normalized propagation constant, whose permitted values range between 0 and 1, the group delay per unit distance of ray x can be written as

$$\tau_\alpha(x)=\frac{n_o}{c}\left[1+A(\alpha)x+B(\alpha)x^2\right] \tag{1}$$

where n_o is the on-axis index, c is the free-space velocity of light, $A(\alpha)$ and $B(\alpha)$ are constants, depending on the profile parameter α. All the intermodal time dispersion properties of a single fiber can be described through the

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complex quantity

$$\bar{P}(\omega)=\int_0^1 p_\alpha(x)\exp[-i\omega\tau_\alpha(x)z]\,dx \quad (2)$$

where ω is the angular modulation frequency, i the imaginary unit, $p_\alpha(x)$ the optical power distribution in the domain of variable x, and finally z is the fiber length. $P_\alpha(x)$ depends on the launching condition and on the profile parameter α. The baseband response of the fiber, due to intermodal time dispersion, can be defined as $\bar{H}(\omega)=\bar{P}(\omega)/P(O)$.

We consider now a pair of cascaded fibers, with F1 indicating the transmitting and F2 the receiving fiber. Our purpose is to determine an equivalent optical power distribution p_e and an equivalent phase distribution ψ_e, so that the baseband response of the optical link consisting of the two fibers can be obtained through a simple expression like (2). Assuming as integration variable the normalized propagation constant of F2, i.e., x_2, in the most general case we can expect to have

$$\bar{P}(\omega)=\int_0^1 p_e(x_2,\omega)\exp[-i\psi_e(x_2,\omega)]\,dx_2. \quad (3)$$

We shall show later that it is possible to obtain this approximate expression

$$\bar{P}(\omega)\cong\int_0^1 p_e(x_2)\exp[-i\omega t_e(x_2)]\,dx_2 \quad (4)$$

in which $t_e(x_2)$ represents an equivalent time delay distribution.

The resemblance between (4) and (2) encourages extending this model to more than two cascaded fibers. Such a method could be very interesting for predicting the baseband response of optical links made of many cascaded fibers, knowing the differential mode delay distributions of the single fibers, their material dispersion properties, and approximately the accuracy of the jointing procedure. This prediction, starting from the single baseband responses, appears questionable, as recent measurements have shown [12]. In the following, we will consider fibers having equal average NA's, core radii, and on-axis indices, in order to stress the role played by the different profile parameters α.

III. Simplified Model of a Joint

In this section, we recall briefly some properties of a single α-profile fiber, then the optical power transfer at the joint will be described. The radiance distribution B, at the input or output section of a fiber, is related to the total guided power P through the quadruple integral

$$P=\int_0^{2\pi}d\theta_\phi\int_0^{2\pi}d\phi\int_0^a r\,dr\int_0^{\theta_{aM}}B(r,\phi,\theta_a,\theta_\phi)\sin\theta_a\cos\theta_a\,d\theta_a \quad (5)$$

where r and ϕ are polar coordinates in the fiber cross section, θ_a is the ray propagation angle in air, θ_ϕ is the angle between the radial direction and the projection of the ray trajectory on the transverse plane. Finally, a is the core radius and θ_{aM} the maximum permitted value of θ_a, imposed by the fiber NA.

For an α-profile fiber, whose refractive index distribution is

$$n(r)=n_0\sqrt{1-2\Delta\left(\frac{r}{a}\right)^\alpha}$$

where all the parameters are known, except for Δ, which imposes the fiber NA, we can define a normalized propagation constant x as

$$x=\frac{1}{2\Delta}\left(1-\beta^2/k_0^2\right),\qquad k_0=\frac{2\pi}{\lambda}n_0 \quad (6)$$

in which λ is the wavelength of light. With this variable, it is possible to simplify expression (5), provided that B, which is imposed by the launching condition, is a suitable function of $r,\phi,\theta_a,\theta_\phi$.

In the Appendix, it is shown that, when B depends only on x, r/a, and ϕ, which is the case of a joint affected by lateral displacement, we can write

$$P=8\pi\Delta n_0^2a^2\int_0^1 dx\int_0^{\pi/2}d\phi\int_0^{x^{1/\alpha}}B\left(x,\frac{r}{a},\phi\right)\left(\frac{r}{a}\right)d\left(\frac{r}{a}\right). \quad (7)$$

Note that, when B depends only on x, by a simple integration, we have

$$P=2\pi^2\Delta n_0^2a^2\int_0^1 B(x)x^{2/\alpha}\,dx. \quad (8)$$

Comparing with (2), in which we have to assume $\omega=0$, one can obtain the following relationship between $B(x)$ and $p_\alpha(x)$:

$$p_\alpha(x)=2\pi^2\Delta n_o^2a^2x^{2/\alpha}B(x). \quad (9)$$

In Fig. 1, the cross sections of the two fibers F1 and F2 are shown at the joint. We have two radial coordinates r_1, r_2, simply related by Carnot's theorem

$$r_1^2=r_2^2+d^2-2r_2d\cos\phi. \quad (10)$$

In the Appendix, it is shown that the variable x defined by (6) can be expressed as a function of the radial coordinate r and of the propagation angle in the fiber θ. It is

$$x=\left(\frac{r}{a}\right)^\alpha\cos^2\theta+\frac{\sin^2\theta}{2\Delta}. \quad (11)$$

At the interface, by a straightforward application of Snell's law, we can write

$$x_1=x_2+\left(\frac{r_1}{a}\right)^{\alpha_1}-\left(\frac{r_2}{a}\right)^{\alpha_2}. \quad (12)$$

Substituting (10) into this equation, and neglecting terms of order greater than one in d/a, we obtain

$$x_1\cong x_2+\left(\frac{r_2}{a}\right)^{\alpha_1}-\alpha_1\left(\frac{r_2}{a}\right)^{\alpha_1-1}\frac{d}{a}\cos\phi-\left(\frac{r_2}{a}\right)^{\alpha_2}. \quad (13)$$

From (7), the complex quantity necessary for the computation of the baseband response of the optical link turns out to be

$$\bar{P}(\omega)=8\pi\Delta n_o^2a^2\int_0^1 dx_2\int_0^{\phi_M(d/a)}d\phi\int_0^{x_2^{1/\alpha_2}}B_1(x_1)\cdot\exp\{-i\omega[\tau_1(x_1)z_1+\tau_2(x_2)z_2]\}\left(\frac{r_2}{a}\right)d\left(\frac{r_2}{a}\right) \quad (14)$$

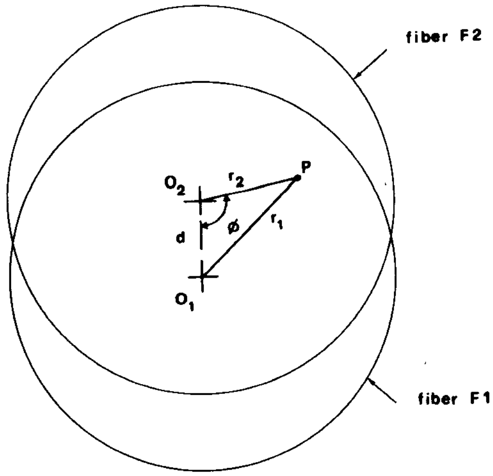

Fig. 1. Cross sections of the two fibers at the joint. O_1 and O_2 are the two fiber centers, P is a generic point.

in which the radiance distribution B_1, at the input of F1, has been assumed to be a function of x_1 only. The upper extreme of ϕ-integration, denoted $\phi_M(d/a)$, is not exactly $\pi/2$, but it is slightly smaller. However, this fact, which is important for joint loss evaluation, can be considered as practically negligible for time dispersion evaluation, so in the following we will assume $\phi_M = \pi/2$. In (14), x_1 is obtained as a function of $x_2, r_2/a, \phi$, from (13).

In order to simplify the analysis, without loss of generality, we consider now $z_1 = z_2 = z$, and a uniform launching condition in F1, so that $B_1(x_1)$ becomes a constant B_o. In this way, we obtain

$$\bar{P}(\omega) = 8\pi\Delta n_o^2 a^2 B_o \int_0^1 \exp\left[-i\omega\tau_2(x_2)z\right] dx_2 \int_0^{\pi/2} d\phi \int_0^{x_2^{1/\alpha_2}} \cdot \exp\left[-i\omega\tau_1\left(x_2, \frac{r_2}{a}, \phi\right)z\right]\left(\frac{r_2}{a}\right) d\left(\frac{r_2}{a}\right). \quad (15)$$

We can assume that, for α_1 close to α_2, and $d/a \ll 1$, x_2 is much greater than the other terms on the right-hand side of (13), so that

$$\tau_1(x_2+\epsilon) \cong \tau_1(x_2) + \tau_1'(x_2)\epsilon$$

$$\tau_1'(x_2) = \frac{n_o}{c}\left[A(\alpha_1) + 2B(\alpha_1)x_2\right]$$

$$\epsilon = \left(\frac{r_2}{a}\right)^{\alpha_1} - \left(\frac{r_2}{a}\right)^{\alpha_2} - \alpha_1\left(\frac{r_2}{a}\right)^{\alpha_1-1}\frac{d}{a}\cos\phi$$

and hence

$$\bar{P}(\omega) = 8\pi\Delta n_o^2 a^2 B_o \int_0^1 \cdot \exp\left\{-i\omega\left[\tau_1(x_2) + \tau_2(x_2)\right]z\right\} dx_2 \int_0^{\pi/2} d\phi \cdot \int_0^{x_2^{1/\alpha_2}} \exp\left[-i\omega\tau_1'(x_2)\epsilon z\right]\left(\frac{r_2}{a}\right) d\left(\frac{r_2}{a}\right). \quad (16)$$

At the modulation frequencies of interest, i.e., not much higher than the 3-dB bandwidth, we can consider that $\omega\tau_1'(x_2)\epsilon z$ is a very small angle, therefore

$$\exp\left[-i\omega\tau_1'(x_2)\epsilon z\right] \cong 1 - i\omega\tau_1'(x_2)\epsilon z.$$

This approximation allows one to express the innermost integral of (16) in a completely analytical form, leading to

$$\bar{P}(\omega) \cong 2\pi^2\Delta n_o^2 a^2 B_o \int_0^1 \cdot \exp\left\{-i\omega\left[\tau_1(x_2) + \tau_2(x_2) + \delta\tau(x_2)\right]z\right\} x_2^{2/\alpha_2}\, dx_2 \quad (17)$$

with

$$\delta\tau(x_2) = \frac{n_o}{c}\left[A(\alpha_1) + 2B(\alpha_1)x_2\right] \cdot \left(\frac{2}{\alpha_1+2}x_2^{\alpha_1/\alpha_2} - \frac{2}{\alpha_2+2}x_2 - \frac{4}{\pi}\frac{\alpha_1}{\alpha_1+1}x_2^{(\alpha_1-1)/\alpha_2}\frac{d}{a}\right).$$

Since $\delta\tau(x_2)$ is small with respect to $\tau_1(x_2)$, it is possible to write

$$\tau_1(x_2) + \delta\tau(x_2) \cong \tau_1(\langle x_1 \rangle)$$

$$\langle x_1 \rangle = x_2 + \frac{2}{\alpha_1+2}x_2^{\alpha_1/\alpha_2} - \frac{2}{\alpha_2+2}x_2 - \frac{4}{\pi}\frac{\alpha_1}{\alpha_1+1}x_2^{(\alpha_1-1)/\alpha_2}\frac{d}{a}. \quad (18)$$

In other words, the time delay distribution of fiber F1 can be added directly to that of fiber F2, provided that x_1 is replaced by the new variable $\langle x_1 \rangle$ given by (18). Finally, comparing with (4), one obtains

$$p_e(x_2) \cong 2\pi^2\Delta n_o^2 a^2 B_o x_2^{2/\alpha_2} \quad (19)$$

which coincides with the optical power distribution under a uniform launching condition for a single fiber, and

$$t_e(x_2) \cong \left[\tau_1(\langle x_1 \rangle) + \tau_2(x_2)\right]z. \quad (20)$$

Note that in (18), two distinct deviations from x_2 are present: one accounts for the index profile variation at the joint, the other is proportional to the joint offset d/a. However, in (17), the main contributions to the time dispersion properties of the optical link consist of $\tau_1(x_2)$ and $\tau_2(x_2)$. Therefore, the well-known compensation effects are to be expected when $\tau_1(x_2)$ and $\tau_2(x_2)$ have approximately equal values and opposite signs.

In order to check the validity of the present model, we show in Fig. 2 a comparison between the baseband responses of the same optical link, calculated exactly from (14) (continuous line), and by approximation (4), (18), (19), and (20) (dashed line). We observe very good agreement, at least for modulation frequencies lower than twice the 3-dB bandwidth. For comparison, the baseband responses of the two single fibers are also plotted in Fig. 2, showing an excellent compensation. Repeated calculations for different values of d/a have pointed out slight variations in this compensation; however the agreement between the results obtained from the two procedures remains very good. Since these results have been obtained for a pair of optimally compensating fibers (the 3-dB bandwidth over the double distance is much greater than those of the two single fibers), it seems to be correct to assume that the approximate method described in this section is always right. Then, in the following sections, we will present results obtained from such a method only.

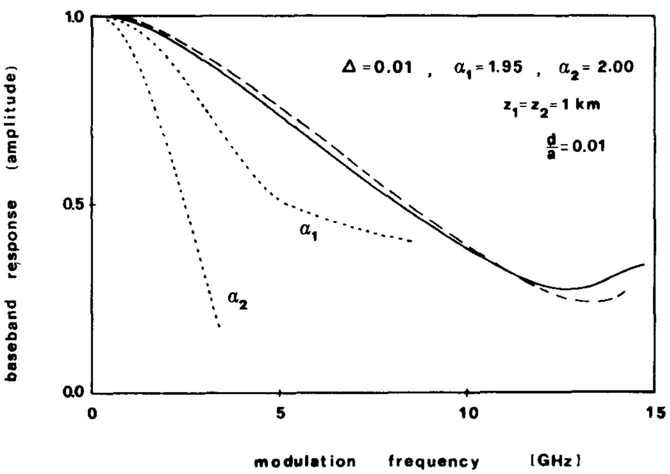

Fig. 2. Comparison between the baseband responses of the same optical link, calculated exactly (continuous line), and approximately (dashed line). In dotted lines the baseband responses of the two single fibers.

IV. Results for Two Fibers

There are different ways for studying the effects of intermodal time dispersion: evaluation of the overall pulse width $\Delta\tau$ [11], of the rms pulse width σ [13], and of the 3-dB bandwidth [14]. We restrict ourselves to the first two methods. For a single fiber, the optimum α to minimize $\Delta\tau$ is $\alpha_{o\Delta} = 2 - 2\Delta$ [11], whereas the optimum α to minimize σ is $\alpha_{o\sigma} = 2 - 12/5\Delta$ [13]. The analytical expression of $t_e(x_2)$, obtained in the previous section for a pair of cascaded fibers, leads to the following approximate relationships:

$$\frac{\alpha_1 + \alpha_2}{2} \cong \alpha_{o\Delta} \qquad \frac{\alpha_1 + \alpha_2}{2} \cong \alpha_{o\sigma} \tag{21}$$

respectively, for the minimization of $\Delta\tau$ and σ of the optical link. The above formulas between the two profile parameters α_1, α_2 have been obtained under the condition $|\alpha_1 - \alpha_2| \ll (\alpha_1 + \alpha_2)$, and ignoring d/a. Nevertheless, for $0 \leqslant d/a \leqslant 0.04$, which is a typical range of values for practical joints, the minima in $\Delta\tau$ and σ shift very little.

In Fig. 3, $\Delta\tau$ and σ are plotted as functions of α_2, with α_1 as a parameter. In this calculation we have assumed $\Delta = 0.01$, $z = 1$ km, $d/a = 0$. The dashed horizontal lines refer to the case of two equal fibers, having $\alpha_1 = \alpha_2 = \alpha_{o\Delta}$ and $\alpha_1 = \alpha_2 = \alpha_{o\sigma}$, respectively. We observe that σ for two different fibers reaches a minimum which is lower than that of two equal fibers, whereas the minima of $\Delta\tau$ lie approximately on the dashed line. The latter behavior is due to the small number of degrees of freedom of $t_e(x_2)$, which is practically a parabola, very similar to $\tau_\alpha(x)z$, except for having $(\alpha_1 + \alpha_2)/2$ instead of the simple α. By contrast, to determine σ, the optical power distribution also plays a significant role, and, for two different fibers, turns out to favor rays whose group delays are closer to each other.

The order of installation of the two fibers affects the values of $\Delta\tau$ and σ obtained, but not in a manner so significant to be distinguishable on graphs like those reported in Fig. 3. This influence is due to the redistribution of the optical power among the rays, which takes place at the joint with different properties in the two directions, owing to the difference between α_1 and α_2. From (18), when $d/a = 0$, for $\alpha_1 < \alpha_2$, we have x_2 smaller than $\langle x_1 \rangle$, i.e., the optical power flows towards the inner rays, whereas for $\alpha_1 > \alpha_2$, x_2 is greater than $\langle x_1 \rangle$, i.e., the optical power flow is directed to the outer rays. For a single fiber, when $\alpha > 2 - 4\Delta$, the outer rays are mainly responsible of a tail in the impulse response. Therefore, for the range of values $(\alpha_1 + \alpha_2)/2 > 2 - 4\Delta$, which includes also $\alpha_{o\Delta}$ and $\alpha_{o\sigma}$, the condition $\alpha_1 < \alpha_2$ allows reaching minimum dispersion.

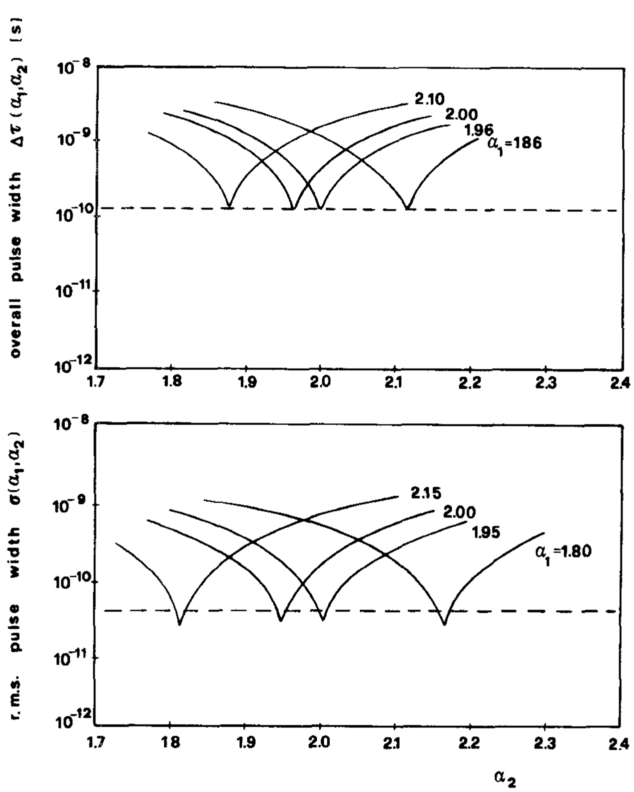

Fig. 3. Overall pulse width and rms pulse width versus α_2 with α_1 as a parameter, for $\Delta = 0.01$, $z_1 = z_2 = 1$ km, and $d/a = 0$.

In Fig. 4, some curves $t_e(x_2)$ are plotted, for different α_1, α_2, confirming substantially that the time delay distribution at the output of the optical link is similar to that of an equivalent pair of equal fibers having $\alpha = (\alpha_1 + \alpha_2)/2$. Also, in this case, we have assumed $d/a = 0$.

The influence of a joint offset can be considered as comparable to that of the order of installation. From (18), when $\alpha_1 = \alpha_2$, the presence of $d/a \neq 0$ leads to x_2 greater than $\langle x_1 \rangle$, and hence an optical flow towards the outer rays. Yet, when $\alpha_1 \neq \alpha_2$ different behaviors may occur, and a precise rule of influence is difficult to infer. However, only variations of the order of a few percent in $\Delta\tau$ and σ can be expected, provided that d/a is smaller than 0.04. Furthermore, since the present model neglects any mode filtering, mathematically expressed by the presence of $\phi_M(d/a)$ instead of $\pi/2$ as upper limit of ϕ-integration in (14), it might be inappropriate to generalize such a behavior. A more detailed analysis of the time dispersion effects of mode filtering due to joint misalignments, but restricted to a pair of equal cascaded fibers, is reported in [8].

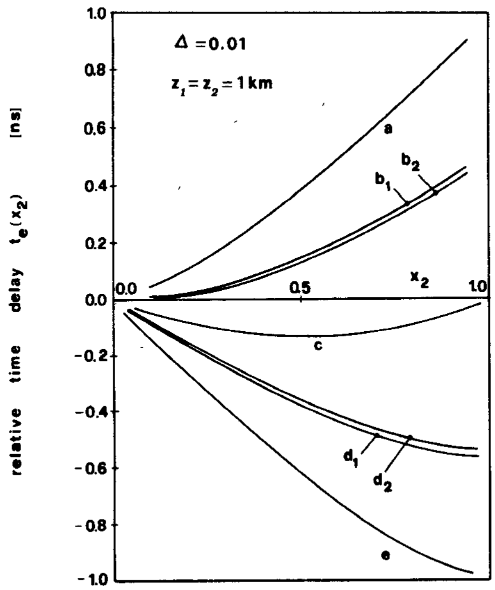

Fig. 4. Relative time delay distributions $t_e(x_2)$, for various pairs $\alpha_1-\alpha_2$. a: 2.02–2.02, b_1: 2.02–1.98, b_2: 1.94–2.06, c: 1.98–1.98, d_1: 2.02–1.90, d_2: 1.94–1.98, e: 1.94–1.94.

V. Results for N Fibers

The case of N cascaded fibers, with profile parameters $\alpha_1, \alpha_2, \cdots, \alpha_N$, separated by $N-1$ joints, whose offsets are $d_1, d_2, \cdots, d_{N-1}$, can be studied with the help of the following recurrence formula:

$$x_{n-1}=x_n+\frac{2}{\alpha_{n-1}+2}x_n^{\alpha_{n-1}/\alpha_n}-\frac{2}{\alpha_n+2}x_n - \frac{4}{\pi}\frac{\alpha_{n-1}}{\alpha_{n-1}+1}x_n^{(\alpha_{n-1}-1)/\alpha_n}\frac{d_{n-1}}{a}, \quad n=N,N-1,\cdots,2 \quad (22)$$

which has been derived from (18). Symbol $\langle\ \rangle$ has been omitted for the sake of simplicity. Finally, instead of (4) and (20), we have

$$\overline{P}(\omega)=\int_0^1 p_e(x_N)\exp\left[-i\omega t_e(x_N)\right]dx_N \quad (23)$$

$$t_e(x_N)=\left[\tau_1(x_1)+\tau_2(x_2)+\cdots+\tau_N(x_N)\right]z. \quad (24)$$

Also in this case, in order to minimize $\Delta\tau$ and σ, the average value of the α_n, $n=1,2,\cdots,N$, must be $\alpha_{o\Delta}$ and $\alpha_{o\sigma}$, respectively. Nevertheless here a broader spread of results occurs, depending on the order of installation and on the joint misalignments.

Assuming 5 fibers, each 1 km long, with perfect joints, whose profile parameters are randomly distributed in a given α-interval, but satisfying the formula

$$\sum_{n=1}^{5}\alpha_n=5\alpha_{o\sigma}$$

there are 120 possible orders of installation, and as many different values of σ. The minimum value of σ obtained $\sigma_{\min}$ corresponds to the condition $\alpha_1\leqslant\alpha_2\leqslant\alpha_3\leqslant\alpha_4\leqslant\alpha_5$, whereas the maximum $\sigma_{\max}$ corresponds to the condition $\alpha_1\geqslant\alpha_2\geqslant\alpha_3\geqslant\alpha_4\geqslant\alpha_5$. In Table I, $\sigma_{\min}$ and $\sigma_{\max}$ for three

TABLE I
Minimum and Maximum rms Pulse width over 5 Jointed Fibers

range of α values	σ_{min} [ps]	σ_{max} [ps]
1.95 - 2.00	48	49
1.93 - 2.02	50	52
1.90 - 2.05	54	58

α-intervals are compared. For a narrower α-interval, a smaller difference between $\sigma_{\max}$ and $\sigma_{\min}$ occurs, and therefore better compensation is achieved.

Assuming 5 fibers, each 1 km long, having $\alpha_1=\alpha_2=\alpha_3=\alpha_4=\alpha_5=2$, and separated by 4 joints whose offsets are randomly distributed between 0 and 0.04, there are 24 possible combinations of offsets, and as many different values of σ. In this case, we have $\sigma_{\min}=361$ ps for $d_1/a\leqslant d_2/a\leqslant d_3/a\leqslant d_4/a$, and $\sigma_{\max}=370$ ps for $d_1/a\geqslant d_2/a\geqslant d_3/a\geqslant d_4/a$.

VI. Discussion

In the present section, first we take into account the effects of intramodal time dispersion, and then we consider a possible extension of our model to profiles not of the α-type. This gives the opportunity for a discussion of the choice of the optical source and of practical uses of compensation effects.

Letting $\tau(\lambda)$ the time delay per unit distance of the chromatic component characterized by the wavelength λ, as a first approximation, it can be written as

$$\tau(\lambda)\cong\tau(\lambda_o)+\tau'(\lambda_o)(\lambda-\lambda_o) \quad (25)$$

where λ_o is the central wavelength of emission of the optical source and $\tau'(\lambda_o)$, called material dispersion, represents the first derivative of $\tau(\lambda)$ at $\lambda=\lambda_o$. If one assumes, for simplicity, a spectral power distribution $p(\lambda)$ which is uniform between $(\lambda_o-\Delta\lambda/2)$ and $(\lambda_o+\Delta\lambda/2)$, and zero out of this interval, it is possible to obtain the following formulas, for the overall pulse width $\Delta\tau_\lambda$ and the rms pulsewidth σ_λ due to intramodal time dispersion

$$\Delta\tau_\lambda=|\tau'(\lambda_o)|\Delta\lambda z \quad \sigma_\lambda=\frac{1}{2\sqrt{3}}|\tau'(\lambda_o)|\Delta\lambda z. \quad (26)$$

These quantities must be compared with $\Delta\tau$ and σ due to intermodal time dispersion, to determine which cause of time dispersion is predominant. The two comparisons may give rather different results, owing to the shape of the intermodal impulse response.

In order to give an idea of the practical use of this analysis, we consider only the former comparison on a pair of fibers, each 1 km long, jointed without offsets. The first fiber is characterized by $\alpha_1=2$. Finally, we assume two different values of material dispersion: $|\tau'(\lambda_o)|=100\text{ps}\cdot(\text{nm}\cdot\text{km})^{-1}$, which is typical when $\lambda=850$–900 nm, and $|\tau'(\lambda_o)|=10$ ps$\cdot(\text{nm}\cdot\text{km})^{-1}$, which is typical when $\lambda=$ 1250–1300 nm. These two intervals of values of λ correspond to the two main minima of fiber spectral loss, which

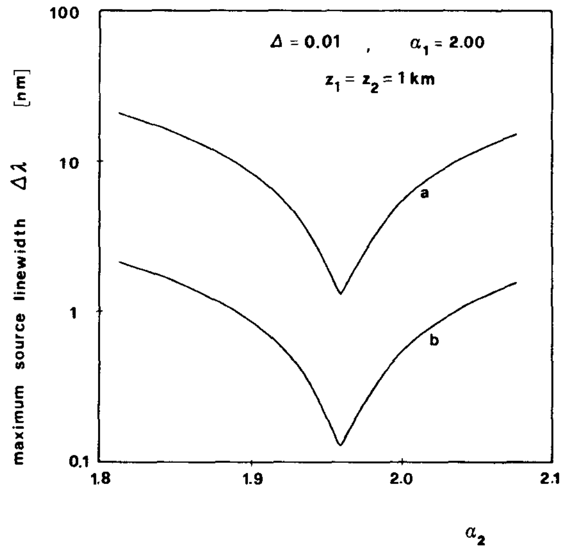

Fig. 5. Maximum source linewidth $\Delta\lambda$ for which $\Delta\tau_\lambda < 0.2\,\Delta\tau$. Curve a for $|\tau'(\lambda_o)| = 10$ ps·(nm·km)$^{-1}$, curve b for $|\tau'(\lambda_o)| = 100$ ps·(nm·km)$^{-1}$.

are called, respectively, the first and the second window. Fig. 5 shows, as a function of the second fiber profile parameter α_2, the maximum $\Delta\lambda$ for which $\Delta\tau_\lambda$ turns out to be smaller than 0.2 $\Delta\tau$, i.e., for maintaining intermodal time dispersion as the dominant cause of the total time dispersion. This condition allows one to exploit almost completely the compensation behavior on intermodal time dispersion of the pair of cascaded fibers, discussed in the previous sections. Otherwise, intramodal time dispersion, which cannot be compensated, begins to add its effects, which become more and more noticeable with increasing $\Delta\lambda$.

We can divide the optical sources into three main classes: single-mode laser diodes, whose $\Delta\lambda$ is smaller than 0.1 nm, multimode laser diodes, characterized by $0.5 \leqslant \Delta\lambda \leqslant 2$ nm, LED's, having $\Delta\lambda \geqslant 10$ nm. Only single-mode laser diodes allow a complete exploitation of intermodal dispersion compensation at any wavelength. Multimode laser diodes can be efficiently employed in the second window only, whereas LED's cannot be employed at all. Since the curves of $\Delta\tau$ versus α_2 with α_1 as a parameter are very similar to each other, except for a shift along the α_2 axis, the above conclusions about the choice of the optical source can be considered as independent of α_1.

When the index profile is not of the α-type, we have to describe the optical power in terms of a pair of mode variables, instead of a single variable [15]. Assuming that τ depends both on the propagation constant β and on the azimuthal mode number ν, we can write, instead of (2), for a single fiber, the following equation:

$$\overline{P}(\omega) = \iint_G p(\beta,\nu)\exp\left[-i\omega\tau(\beta,\nu)z\right]\,d\beta\,d\nu \qquad (27)$$

where G denotes the domain of guided rays, and $p(\beta,\nu)$ is the optical power distribution in that domain, which depends on the particular shape of the index profile. Nevertheless, (4) for a pair of cascaded fibers can be replaced by

$$\overline{P}(\omega) \cong \iint_G p_e(\beta_2,\nu_2)\exp\left[-i\omega t_e(\beta_2,\nu_2)\right]\,d\beta_2\,d\nu_2 \qquad (28)$$

in which $p_e(\beta_2,\nu_2)$ and $t_e(\beta_2,\nu_2)$ can be derived from the knowledge of the two index profiles and of the joint offset.

If the index profiles are not too different and if the joint offset is small, for $z_1 = z_2 = z$, we can certainly write an approximate equation of the type

$$t_e(\beta_2,\nu_2) \cong \left[\tau_1(\langle\beta_1\rangle,\langle\nu_1\rangle) + \tau_2(\beta_2,\nu_2)\right]z \qquad (29)$$

where

$$\langle\beta_1\rangle = \beta_1 + \delta\beta_1\left(\Delta n, \frac{d}{a}\right), \qquad \langle\nu_1\rangle = \nu_1 + \delta\nu_1\left(\Delta n, \frac{d}{a}\right) \qquad (30)$$

in which $\delta\beta_1$ and $\delta\mu_1$ represent small corrections with respect to β_1 and ν_1, and Δn takes formally into account the difference between the two index profiles. $\delta\beta_1(\Delta n, d/a)$ and $\delta\nu_1(\Delta n, d/a)$ can be obtained from geometrical considerations, which are conceptually as simple as those for α-profile fibers. Only the numerical computation appears more difficult, because of the double integrals over the domain G. However, since $\delta\beta_1$ and $\delta\nu_1$ are small with respect to β_1 and ν_1, one can expect, as for α-profile fibers, that the best compensation occurs when $\tau_1(\beta_2,\nu_2)$ and $\tau_2(\beta_2,\nu_2)$ have approximately equal values and opposite signs.

In conclusion, the model presented in the previous sections should be assumed only as an explanatory example, in order to have simple rules of compensation. Yet, for real fibers, which never are rigorously of the α-type, it is more correct to consider a general equation, like (29). $\tau(\beta,\nu)$ can be experimentally determined by a differential mode delay measurement. This measurement can be performed either selecting modes at the fiber input [16], or selecting modes at the fiber output [17]. An output selection allows usually a better sensitivity, and appears suitable also for testing a set of cascaded fibers, in order to check intermodal compensation.

Before concluding this discussion, we will stress the fundamental role played by the assumption that mode coupling has negligible effects, which is at the basis of our model. Recently [18], it has been shown that a remarkable degree of mode coupling may be present among modes having the same β, whereas a very weak coupling characterizes modes with different β. This has no consequence for the study of α-profile fibers, in which τ_α is a function of β only. Yet, considering a profile not of the α-type, for which τ depends also on ν, an extended model could be sometimes preferable, which takes into account mode coupling too, at least within groups of degenerate modes. This will be done in a subsequent paper.

VII. Conclusions

Intermodal time dispersion can be compensated in cascaded multimode fiber links, installing fibers whose profiles produce time delay distributions of opposite signs.

Yet, intramodal time dispersion fixes a lower limit to such a compensation, which increases with increasing spectral width of the optical source employed. These conclusions are not limited to α-profile fibers, although for this class of fibers a quantitative evaluation of the compensation appears simpler. This is due to the fundamental role played by the unique profile parameter α, which allows one to determine explicit compensation formulas, like (21). Nevertheless, for any class of fibers, it is possible to attain a practical intermodal compensation, on the basis of experimental measurements performed on the single fibers, before jointing. These measurements, which consist of differential mode delay investigations, can be repeated after the installation, in order to check the compensation behavior.

Appendix

The expression of the propagation constant is

$$\beta = \frac{2\pi}{\lambda} n(r) \cos\theta$$

where θ is the propagation angle in the fiber. Substituting into (6), we have

$$x = \left(\frac{r}{a}\right)^{\alpha} \cos^2\theta + \frac{\sin^2\theta}{2\Delta}.$$

The maximum permitted value of θ_a, θ_{aM}, is given by

$$\sin^2\theta_{aM} = n^2(r) - n^2(a)$$

so that (5), assuming B as independent of θ_ϕ, can be rewitten as

$$P = \pi \int_0^{2\pi} d\phi \int_0^a r\,dr \int_0^{n_o^2 2\Delta[1-(r/a)^\alpha]} B(r, \phi, \sin^2\theta_a)\, d(\sin^2\theta_a).$$

Replacing variable $\sin^2\theta_a$ with x, through the above expression of x, and neglecting terms of order greater than one in Δ, we are left with

$$P = 8\pi\Delta n_o^2 a^2 \int_0^{\pi/2} d\phi \int_0^1 \left(\frac{r}{a}\right) d\left(\frac{r}{a}\right) \int_{(r/a)^\alpha}^1 B\left(x, \frac{r}{a}, \phi\right) dx$$

and by an inversion of the order of integration

$$P = 8\pi\Delta n_o^2 a^2 \int_0^{\pi/2} d\phi \int_0^1 dx \int_0^{x^{1/\alpha}} B\left(x, \frac{r}{a}, \phi\right)\left(\frac{r}{a}\right) d\left(\frac{r}{a}\right).$$

References

[1] M. Eve, "Multipath time dispersion theory of optical network," *Opt. Quant. Electron.*, vol. 10, pp. 41–51, Jan. 1978.

[2] J. V. Wright, M. C. Brierley, and C. J. Todd, "Performance prediction of multimode fibre links," in *Proc. 6th E.C.O.C.*, York, U. K., 1980, pp. 378–381.

[3] A. B. Puc and G. L. Yip, "Pulse broadening compensation in fibre-optical links," *Electron. Lett.*, vol. 16, pp. 162–163, Feb. 28, 1980.

[4] P. Galeati, V. Rizzoli, and C. G. Someda, "Bandwidth behaviour in tandem-connected graded-index multimode fibers," *J. Opt. Comm.*, vol. 2, pp. 54–58, June 1981.

[5] T. Tanifuji, "Exact analysis of intermodal dispersion compensation in spliced graded-index optical fibers," *IEEE J. Quant. Electron.*, vol. QE-18, pp. 844–849, May 1982.

[6] R. Olshansky, "Mode coupling effects in graded-index optical fibers," *Appl. Opt.*, vol. 14, pp. 935–945, Apr. 1975.

[7] G. Cancellieri and P. Fantini, "Mode coupling effects in optical fibres: perturbative solution of the time-dependent power flow equation," *Opt. Quant. Electron.*, vol. 15, pp. 119–136, Mar. 1983.

[8] ——, "Time dispersion effects in optical fiber joints," *Opt. Comm.*, vol. 45, no. 1, pp. 30–34, Mar. 1983.

[9] P. Di Vita and U. Rossi, "Theory of power coupling between multimode optical fibres," *Opt. Quant. Electron.*, vol. 10, pp. 107–117, Mar. 1978.

[10] D. Gloge, "Offset and tilt loss in optical fiber splices," *Bell Syst. Tech. J.*, vol. 55, pp. 905–916, Sept. 1976.

[11] D. Gloge and E. A. J. Marcatili, "Multimode theory of graded-core fibers," *Bell Syst. Tech. J.*, vol. 52, pp. 1963–1978, Nov. 1973.

[12] J. V. Wright and B. P. Nelson, "Bandwidth studies of concatenated multimode fibre links," in *Proc. Symp. Opt. Fiber Meas.* (Boulder, CO), Oct. 13–14, 1982, pp. 9–12.

[13] R. Olshansky and D. B. Keck, "Pulse broadening in graded-index optical fibers," *Appl. Opt.*, vol. 15, pp. 483–491, Feb. 1976.

[14] D. Marcuse, "Calculation of bandwidth from index profiles of optical fibers. 1: Theory," *Appl. Opt.*, vol. 18, pp. 2073–2080, June 15, 1979.

[15] E. A. J. Marcatili, "Modal dispersion in optical fibers with arbitrary numerical aperture and profile dispersion," *Bell Syst. Tech. J.*, vol. 56, pp. 49–63, Jan. 1977.

[16] R. Olshansky and S. M. Oaks, "Differential mode delay measurement," in *Proc. 4th E.C.O.C.* (Geneva), Sept. 12–15, 1978, p. 128–132.

[17] G. Cancellieri, M. Mezzetti, C. G. Someda, and M. Zoboli, "Simplified procedure for indirect tests of optical-fibre transfer function," *Electron. Lett.*, vol. 15, pp. 234–236, Apr. 12, 1979.

[18] S. Kawakami, and N. Tanji, "Evolution of power distribution in graded-index fibers," in *Proc. IOCC 1983* (Tokyo), June 27–30, 1983, pp. 24–27.

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